Lesson 2.1 • Inductive Reasoning

Name ___________________________ Period ___________ Date ________________

For Exercises 1–7, use inductive reasoning to find the next two terms in each sequence.

1. 4, 8, 12, 16, _____, _____
2. 400, 200, 100, 50, 25, _____, _____
3. \[
\begin{array}{c}
\frac{1}{8}, \frac{2}{7}, \frac{1}{4}, \frac{5}{2} \end{array}
\] _____, _____
4. \(-5, 3, -2, 1, -1, 0, _____, _____
5. 360, 180, 120, 90, _____, _____
6. 1, 3, 9, 27, 81, _____, _____
7. 1, 5, 14, 30, 55, _____, _____

For Exercises 8–10, use inductive reasoning to draw the next two shapes in each picture pattern.

8. [Diagram]

9. [Diagram]

10. [Diagram]

For Exercises 11–13, use inductive reasoning to test each conjecture. Decide if the conjecture seems true or false. If it seems false, give a counterexample.

11. The square of a number is larger than the number.

12. Every multiple of 11 is a “palindrome,” that is, a number that reads the same forward and backward.

13. The difference of two consecutive square numbers is an odd number.
Lesson 2.2 • Finding the \( n \)th Term

For Exercises 1–4, tell whether the rule is a linear function.

1. \[
\begin{array}{c|cccc}
\( n \) & 1 & 2 & 3 & 4 \\
\( f(n) \) & 8 & 15 & 22 & 29
\end{array}
\]

2. \[
\begin{array}{c|cccc}
\( n \) & 1 & 2 & 3 & 4 \\
\( g(n) \) & 14 & 11 & 8 & 5
\end{array}
\]

3. \[
\begin{array}{c|cccc}
\( n \) & 1 & 2 & 3 & 4 \\
\( h(n) \) & -9 & -6 & -2 & 3
\end{array}
\]

4. \[
\begin{array}{c|cccc}
\( n \) & 1 & 2 & 3 & 4 \\
\( j(n) \) & \(-\frac{3}{2}\) & -1 & -\frac{1}{2} & 0
\end{array}
\]

For Exercises 5 and 6, complete each table.

5. \[
\begin{array}{c|cccc}
\( n \) & 1 & 2 & 3 & 4 \\
\( f(n) = 7n - 12 \) & 1 & 2 & 3 & 4
\end{array}
\]

6. \[
\begin{array}{c|cccc}
\( n \) & 1 & 2 & 3 & 4 \\
\( g(n) = -8n - 2 \) & 1 & 2 & 3 & 4
\end{array}
\]

For Exercises 7–9, find the function rule for each sequence. Then find the 50th term in the sequence.

7. \[
\begin{array}{c|cccccccc}
\( n \) & 1 & 2 & 3 & 4 & 5 & 6 & \ldots & \ldots \\
\( f(n) \) & 9 & 13 & 17 & 21 & 25 & 29 & \ldots & \ldots \\
\end{array}
\]

8. \[
\begin{array}{c|cccccccc}
\( n \) & 1 & 2 & 3 & 4 & 5 & 6 & \ldots & \ldots \\
\( g(n) \) & 6 & 1 & -4 & -9 & -14 & -19 & \ldots & \ldots \\
\end{array}
\]

9. \[
\begin{array}{c|cccccccc}
\( n \) & 1 & 2 & 3 & 4 & 5 & 6 & \ldots & \ldots \\
\( h(n) \) & 6.5 & 7 & 7.5 & 8 & 8.5 & 9 & \ldots & \ldots \\
\end{array}
\]

10. Use the figures to complete the table.

11. Use the figures above to complete the table. Assume that the area of the first figure is 1 square unit.

\[
\begin{array}{c|ccccc}
\( n \) & 1 & 2 & 3 & 4 & 5 & \ldots & \ldots & \ldots & \ldots \\
\hline
\text{Number of triangles} & 1 & 5 & 9 & \ldots & \ldots \\
\end{array}
\]

\[
\begin{array}{c|cccccccc}
\( n \) & 1 & 2 & 3 & 4 & 5 & \ldots & \ldots & \ldots & \ldots \\
\hline
\text{Area of figure} & 1 & 4 & 16 & \ldots & \ldots & \ldots & \ldots & \ldots \\
\end{array}
\]
Lesson 2.3 • Mathematical Modeling

1. Draw the next figure in this pattern.
   a. How many small squares will there be in the 10th figure?
   b. How many in the 25th figure?
   c. What is the general function rule for this pattern?

2. If you toss a coin, you will get a head or a tail. Copy and complete the geometric model to show all possible results of three consecutive tosses.
   a. How many sequences of results are possible?
   b. How many sequences have exactly one tail?
   c. Assuming a head or a tail is equally likely, what is the probability of getting exactly one tail in three tosses?

3. If there are 12 people sitting at a round table, how many different pairs of people can have conversations during dinner, assuming they can all talk to each other? What geometric figure can you use to model this situation?

4. Tournament games and results are often displayed using a geometric model. Two examples are shown below. Sketch a geometric model for a tournament involving 5 teams and a tournament involving 6 teams. Each team must have the same chance to win. Try to have as few games as possible in each tournament. Show the total number of games in each tournament. Name the teams a, b, c . . . and number the games 1, 2, 3 . . . .

   a. 3 teams, 3 games (round robin)

   b. 4 teams, 3 games (single elimination)
Lesson 2.4 • Deductive Reasoning

1. \( \triangle ABC \) is equilateral. Is \( \triangle ABD \) equilateral? Explain your answer. What type of reasoning, inductive or deductive, do you use when solving this problem?

2. \( \angle A \) and \( \angle D \) are complementary. \( \angle A \) and \( \angle E \) are supplementary. What can you conclude about \( \angle D \) and \( \angle E \)? Explain your answer. What type of reasoning, inductive or deductive, do you use when solving this problem?

3. Which figures in the last group are whatnots? What type of reasoning, inductive or deductive, do you use when solving this problem?

4. Solve each equation for \( x \). Give a reason for each step in the process. What type of reasoning, inductive or deductive, do you use when solving these problems?
   a. \( 4x + 3(2 - x) = 8 - 2x \)
   b. \( \frac{19 - 2(3x - 1)}{5} = x + 2 \)

5. A sequence begins \(-4, 1, 6, 11 \ldots\)
   a. Give the next two terms in the sequence. What type of reasoning, inductive or deductive, do you use when solving this problem?
   b. Find a rule that generates the sequence. Then give the 50th term in the sequence. What type of reasoning, inductive or deductive, do you use when solving this problem?
Lesson 2.5 • Angle Relationships

For Exercises 1–6, find each lettered angle measure without using a protractor.

1. 

2. 

3. 

4. 

5. 

6. 

For Exercises 7–10, tell whether each statement is always (A), sometimes (S), or never (N) true.

7. _____ The sum of the measures of two acute angles equals the measure of an obtuse angle.

8. _____ If \( \angle XAY \) and \( \angle PAQ \) are vertical angles, then either \( X, A, \) and \( P \) or \( X, A, \) and \( Q \) are collinear.

9. _____ If two angles form a linear pair, then they are complementary.

10. _____ If a statement is true, then its converse is true.

For Exercises 11–15, fill in each blank to make a true statement.

11. If one angle of a linear pair is obtuse, then the other is ____________.

12. If \( \angle A \equiv \angle B \) and the supplement of \( \angle B \) has measure 22°, then \( m\angle A = \) ________________.

13. If \( \angle P \) is a right angle and \( \angle P \) and \( \angle Q \) form a linear pair, then \( m\angle Q = \) ________________.

14. If \( \angle S \) and \( \angle T \) are complementary and \( \angle T \) and \( \angle U \) are supplementary, then \( \angle U \) is a(n) ________________ angle.

15. Switching the “if” and “then” parts of a statement changes the statement to its ________________.
Lesson 2.6 • Special Angles on Parallel Lines

For Exercises 1–3, use your conjectures to find each angle measure.

1. \[
\begin{align*}
\angle a &= 54° \\
\angle b &= 34° \\
\angle c &= 118° \\
\angle d &= 62° \\
\end{align*}
\]

2. \[
\begin{align*}
\angle a &= 65° \\
\angle b &= 20° \\
\angle c &= 78° \\
\angle d &= 44° \\
\end{align*}
\]

3. \[
\begin{align*}
\angle a &= 54° \\
\angle b &= 34° \\
\end{align*}
\]

For Exercises 4–6, use your conjectures to determine whether \( \ell_1 \parallel \ell_2 \), and explain why. If not enough information is given, write “cannot be determined.”

4. \[
\begin{align*}
\angle 1 &= 118° \\
\angle 2 &= 62° \\
\end{align*}
\]

5. \[
\begin{align*}
\angle 1 &= 95° \\
\angle 2 &= 25° \\
\end{align*}
\]

6. \[
\begin{align*}
\angle 1 &= 48° \\
\angle 2 &= 48° \\
\end{align*}
\]

7. Find each angle measure.

8. Find \( x \).

9. Find \( x \) and \( y \).
11. Kite

**LESSON 1.8 • Space Geometry**

1. Rectangular prism
2. Pentagonal prism
3. Rectangular prism
4. Pentagonal prism

5. 18 cubes
6. $x = 2, y = 1$

**LESSON 1.9 • A Picture Is Worth a Thousand Words**

1. Possible locations

2. Dora, Ellen, Charles, Anica, Fred, Bruce

3. Triangles
   - Acute triangles
   - Isosceles triangles
   - Scalene triangles

**LESSON 2.1 • Inductive Reasoning**

1. $20, 24$
2. $12\frac{1}{2}, 6\frac{1}{4}$
3. $5\frac{1}{4}, 2$

4. Possible answers:
   - a. 
   - b. 
   - c. 

5. $-1, -1$
6. $72, 60$
7. $243, 729$
8. $91, 140$

**LESSON 2.2 • Finding the $n$th Term**

1. Linear
2. Linear
3. Not linear
4. Linear
5. $n \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$
   
   \[
   f(n) \quad -5 \quad 2 \quad 9 \quad 16 \quad 23
   \]

6. $n \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$
   
   \[
   g(n) \quad -10 \quad -18 \quad -26 \quad -34 \quad -42
   \]

7. $f(n) = 4n + 5; f(50) = 205$
8. $f(n) = -5n + 11; f(50) = -239$
9. $f(n) = \frac{1}{2}n + 6; f(50) = 31$
10. 

<table>
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<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>...</th>
<th>n</th>
<th>...</th>
<th>50</th>
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<td>5</td>
<td>9</td>
<td>13</td>
<td>17</td>
<td>...</td>
<td>4n-3</td>
<td>...</td>
<td>197</td>
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11. 

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<th>5</th>
<th>...</th>
<th>n</th>
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<tbody>
<tr>
<td>Area</td>
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<td>16</td>
<td>64</td>
<td>256</td>
<td>...</td>
<td>4^n-1</td>
<td>...</td>
<td>4^9</td>
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</table>

**LESSON 2.3 • Mathematical Modeling**

1. 

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   a. 240
   b. 1350
   c. \(f(n) = 2n(n + 2)\), or \(f(n) = 2n^2 + 4n\)

2. 

\[
\begin{array}{cccc}
H & HHH & H & HN

H & HN & H & HTH

H & HTH & H & HTT

H & HTT & H & TTH

H & TTH & H & TTT

T & TTH & T & TTT

T & TTT & T & TTT
\end{array}
\]

   a. 8 sequences
   b. 3 sequences have 1 tail.
   c. \(\frac{3}{8}\)

3. 66 different pairs. Use a dodecagon showing sides and diagonals.

**LESSON 2.4 • Deductive Reasoning**

1. No. Explanations will vary. Sample explanation:
   - Because \(\triangle ABC\) is equilateral, \(AB = BC\). Because \(C\) lies between \(B\) and \(D\), \(BD > BC\), so \(BD\) is not equal to \(AB\). Thus \(\triangle ABD\) is not equilateral, by deductive reasoning.

2. Answers will vary. \(m\angle E > m\angle D\) \((m\angle E = m\angle D + 90^\circ)\); deductive

3. a, e, f; inductive

4. Deductive

   a. \(4x + 3(2 - x) = 8 - 2x\) \(\text{The original equation.}\)

   \[4x + 6 - 3x = 8 - 2x\] \(\text{Distributive property.}\)

   \[x + 6 = 8 - 2x\] \(\text{Combining like terms.}\)

   \[3x + 6 = 8\] \(\text{Addition property of equality.}\)

   \[3x = 2\] \(\text{Subtraction property of equality.}\)

   \[x = \frac{2}{3}\] \(\text{Division property of equality.}\)
Chapter 3: Skills Practice

LESSON 3.1 • Duplicating Segments and Angles

1. \( xy = \frac{19 - 2(3x - 1)}{5} = x + 2 \) 
   The original equation.
   \( 19 - 2(3x - 1) = 5(x + 2) \) 
   Multiplication property of equality.
   \( 19 - 6x + 2 = 5x + 10 \) 
   Distributive property.
   \( 21 - 6x = 5x + 10 \) 
   Combining like terms.
   \( 21 = 11x + 10 \) 
   Addition property of equality.
   \( 11 = 11x \) 
   Subtraction property of equality.
   \( 1 = x \) 
   Division property of equality.

5. a. 16, 21; inductive
   b. \( f(n) = 5n - 9; 241; \) deductive

LESSON 2.5 • Angle Relationships

1. \( a = 68^\circ, b = 112^\circ, c = 68^\circ \)
2. \( a = 127^\circ \)
3. \( a = 35^\circ, b = 40^\circ, c = 35^\circ, d = 70^\circ \)
4. \( a = 90^\circ, b = 90^\circ, c = 42^\circ, d = 48^\circ, e = 132^\circ \)
5. \( a = 20^\circ, b = 70^\circ, c = 20^\circ, d = 70^\circ, e = 110^\circ \)
6. \( a = 70^\circ, b = 55^\circ, c = 25^\circ \)
7. Sometimes
8. Always
9. Never
10. Sometimes
11. acute
12. 158°
13. 90°
14. obtuse
15. converse

LESSON 2.6 • Special Angles on Parallel Lines

1. \( a = 54^\circ, b = 54^\circ, c = 54^\circ \)
2. \( a = 115^\circ, b = 65^\circ, c = 115^\circ, d = 65^\circ \)
3. \( a = 72^\circ, b = 126^\circ \)
4. \( \ell_1 \parallel \ell_2 \)
5. \( \ell_1 \parallel \ell_2 \)
6. cannot be determined
7. \( a = 102^\circ, b = 78^\circ, c = 58^\circ, d = 122^\circ, e = 26^\circ, f = 58^\circ \)
8. \( x = 80^\circ \)
9. \( x = 20^\circ, y = 25^\circ \)