Content Summary

In Chapter 0, students make connections between geometric ideas and things they’ve seen before: common shapes (circles, hexagons, pentagons), mirror symmetry (or reflectional symmetry), and lines and angles. Students study mathematics through art, which may be a new and motivating context for them. This approach is designed to connect to and expand the intuitive understanding they already have about shapes as a whole. In addition to reviewing familiar mathematical ideas, Chapter 0 introduces rotational symmetry, compass and straightedge constructions, and tessellations.

Your student’s teacher may select lessons from Chapter 0 to review skills and ideas that this class needs to understand better. The teacher can also use Chapter 0 as a gentle introduction to Discovering Geometry’s methods for bringing about deep understanding through investigating, working in groups, and using geometry tools and perhaps software. Meanwhile, you can use this time to help establish patterns in the way you’ll work with your student.

Rotational Symmetry

Most people say that a figure is symmetric if it has mirror symmetry—one side is a reflection of the other. There are other kinds of symmetry as well. Chapter 0 introduces rotational symmetry. A figure has 2-fold rotational symmetry if it looks the same after you turn it through half of a full turn—that is, through 180 degrees. It has 3-fold rotational symmetry if it looks the same after being turned one-third of a full turn—that is, through 120 degrees. And so on. The figure on the right, for example, has 3-fold rotational symmetry. All figures look the same after you turn them 360 degrees, so a figure that only satisfies this requirement is not considered to have rotational symmetry.

Students will revisit reflectional and rotational symmetry in the context of transformations in Chapter 7.

Tessellations

If you can make a bunch of tiles all the same shape and use those tiles to cover a flat surface, with no gaps, then you have a tessellation, or tiling pattern. In Chapter 0, students see a few tessellations. In Chapter 7, they’ll study properties of shapes that can be used in tessellations, as well as tessellations made with more than one shape.

Constructions

Mathematicians in ancient Greece believed that the most perfect shapes were circles and straight lines, so they set out to see what they could do with a compass (to make circles) and a straightedge (to make lines). For a compass, they needed a rope with a stake tied to one end. The straightedge was unmarked, so it couldn’t be used for measuring. Nevertheless, they found that with these tools they could construct many shapes and angles. Whatever can be drawn with these tools is called a geometric construction.

Although most of us don’t carry the same idea of perfect shapes as the Greeks did 2500 years ago, studying geometric constructions is valuable for students. It gives them a feeling for shapes and relationships. This feeling is very useful in studying geometric concepts and in logical reasoning.
Chapter 0 • Geometric Art (continued)

Summary Problem
You and your student might discuss this summary problem from Chapter 0. It’s a good problem to revisit several times while working through the chapter.

What ideas from this chapter do you see in Hot Blocks, the fine art pictured on page 24, also shown below?

Discuss these questions with your student from the point of view of a student to your student:

● What kinds of symmetry does the picture have?
● What kinds of symmetry does each block have?
● Could you construct one of the blocks with straightedge and compass?
● What did the artist do to make each block appear three-dimensional?
● Could you construct a different but similar drawing with straightedge and compass?
● Is your own drawing more or less elegant than this one?
● Is it okay to discuss elegance in math?

Some of these questions have several possible valid answers. It is not important that you know all the answers. Instead, as you talk about the answers, make sure your student gives a good explanation of why an answer is reasonable. For example, an answer of “yes” or “no” is not enough. Encourage your student to ask questions too.

Sample Answers
If shading is ignored, the art has 2-fold rotational symmetry, with a vertical line of symmetry through the middle column of blocks and a horizontal line of symmetry between the middle rows of blocks. Each block, if shading is ignored, has 3-fold rotational symmetry and three lines of reflection. The patterns of shading make it appear three-dimensional. The two hexagons that make up each block can be constructed: Starting with circles and using their radii to mark off six equal arcs on the circles, points can be determined to draw equilateral hexagons from which to make blocks.

Although elegance is a matter of opinion, it is acceptable to talk about elegance in any discipline. Here the drawing could be considered elegant because of its artistic value or because of the mathematics it displays.
Chapter 0 • Review Exercises

1. (Lesson 0.1) Which of the designs below have reflectional symmetry? Draw the lines of symmetry. Which of the designs have rotational symmetry?

   ![Designs](image.png)

2. (Lesson 0.2) Name the basic tools of geometry. What are these tools used for?

3. (Lesson 0.3) Use your compass to create a 6-petal daisy design. Color it so it has reflectional symmetry but no rotational symmetry.

4. (Lesson 0.4) Use your straightedge and a copy of your daisy design from Exercise 3 to construct a regular hexagon. Use this to create one block from the Amish quilt tumbling block design pictured on page 14.
2. The compass is used for constructing circles and marking off equal distances, and the straightedge is used for drawing straight lines.

3. Step 1

4. Step 2

Rotational and reflectional

Rotational and reflectional

Rotational

Reflectional